Open University of Cyprus

Faculty of Pure and Applied Sciences

Master of Science in

Sustainable Energy Systems

Master Thesis

Numerical Application for the Calculation of Thermal Transmittance of Building Elements

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Academic Advisor
Paris Fokaides

December 2019

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Summary

In this master thesis, six building elements, three external walls, two roofs and a column consisting of a number of layers of different materials were studied. All the building elements were studied as plane surfaces consisting of thermally homogeneous materials and heat transfer was always steady and one-dimensional, perpendicular to the elements. The purpose of this master thesis was to develop a numerical application for the calculation of thermal transmittance of building elements and to study the effect of thermal insulation on the heat transfer rate through those elements. Heat transfer is not only affected by insulation but also depends on the selection of the materials and their thickness due to the different value of their thermal conductivity. Firstly their overall heat transfer coefficient was calculated with a code written in Matlab without any insulation. Then, their thermal transmittance was calculated with the same code after including insulation and while gradually increasing its thickness. The dependence of the thermal transmittance of the building elements on the insulation thickness was presented graphically and showed that the overall heat transfer coefficient dropped asymptotically with the increase of the thickness of insulation. At last, an effort was done to estimate the optimum insulation thickness for each building element even though for a plane surface the overall heat transfer coefficient continues to decrease with the increase of the thickness of insulation.

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Chapter 1
Introduction

One of the most important aspects in building sector is heat transfer, not only between the building and the natural environment but also inside the building as it directly affects the temperature at the various spaces of the building and also the level of the thermal comfort of the users. Heat is transferred from the environment to a building and vice versa in a complex manner and is greatly affected by environmental conditions such as environmental and ambient temperature, air speed and direction and humidity. Modern buildings are constructed to be as less energy consuming as possible and some can even be categorised as zero-energy buildings or nearly zero-energy buildings if they are combined with renewable energy production close to the building.

The position and the inclination of a surface in relation to the direction of air, the surface geometry and the surface temperature are also of great importance on how the heat will be transmitted through. Specifically for a building, heat is transferred differently between the various building elements because heat flows upwards to the roof, horizontally to a wall and downwards to the floor. Also, the heat transferred through a surface will be different if the surface is perpendicular or horizontal to the heat flow or if it is tilted to an angle. The simpler form of a surface to study heat transfer is the plane wall and subsequently more detailed and advanced analysis needs to be done for more complex geometries like cylinders, spheres and other curved surfaces.

Heat always flows in the direction of temperature gradient, from a medium with higher temperature to a medium with lower temperature. The process continues until the two mediums reach thermal equilibrium state where both of them will have the same temperature. As a result, buildings tend to become hotter during summer and colder during the winter season because they are greatly affected by the weather conditions on the surroundings. For this reason, it is important to prevent any energy losses from the building that would greatly affect the ambient temperature. Also, it is crucial to make a good selection of the materials that will be used during the construction of all the building components.

During last decades it has become even more important to use insulation in buildings so as to seal the building envelope better, in order to decrease any loss of energy as much as possible. Insulation decreases heat exchange between the building and the environment thus maintaining a constant temperature in the interior becomes more feasible without heavy use of equipment (air-conditions, central heat transfer unit etc). There is a great variety of insulating materials in the market like polystyrene, mineral wool, polyurethane foam, urea-formaldehyde foam, polyethylene foam, fiberglass panels, wood panels, perlite, brick, glass and so on. The challenge during the construction of a building though, is to choose, well in advance, the correct insulating material for a particular building element and also, to decide what is the optimum thickness of the insulation for each case according to the use of the building and its energy needs. It can actually become a difficult task given that the inclusion of insulation in a structure can raise the costs significantly. Nevertheless, it is much more convenient and cost-effective to use insulation prior to the construction of a building rather than afterwards with a renovation.

Heat is transferred in three modes, conduction, convection and radiation. Conduction is the heat transfer occurring within a medium from one boundary to the other or between two mediums in direct contact due to actual but invisible molecule movement (mass movement) and happens in solids. Convection is the heat transfer between a solid medium and a fluid. It is related to the energy transferred due to the movement of the mass of a fluid in space through the motion of the molecules. Radiation is the energy transferred in space by electromagnetic radiation. As it is already known, no medium is needed for electromagnetic waves to travel so heat flows from a thermal source to a space without the presence of any matter. (Tipperary Energy Agency)

For the purpose of this master thesis, a numerical application has been developed for the calculation of thermal transmittance of various building elements which are exposed to the environmental conditions, roofs and external walls in particular. The structure of each element is different from one another and subsequently it affects the evaluation of the amount of the heat transferred through the building element. The calculation is simplified by "breaking" each element to its components and then the final result is the sum of the individual contributions. The effect of including insulation in the construction to decrease heat transfer is also studied and then it is examined how the increase of insulation in a plane surface acts on the heat transfer rate. Since all the studied building elements are in direct contact with air internally and externally and there is no information on the boundary conditions the internal and external surface resistances are used to account for the convection and radiation effects. (Tipperary Energy Agency)

Chapter 2
Theoretical Background

In this chapter, initially the three modes of heat transfer, conduction, convection and radiation, are presented and emphasis is given on heat conduction in a plane wall. The concept of thermal resistance is analysed and then it is described how a thermal resistance network can be used for the representation of heat transfer on homogeneous or multilayer plane walls. Also the calculation of thermal values according to ISO 6946 and ISO 10456 is briefly explained. Finally a reference is done on the Directive 2010/31/EU about the energy performance of buildings and particularly Article 5.

2.1 Heat conduction in a plane wall

In order to study head conduction through a plane wall, we assume that steady conditions apply and there is heat transfer only from the one surface of the wall to the other in normal direction to the wall surface. Therefore the heat transfer is one-dimensional since there is no heat transfer and subsequently no temperature change between top and bottom surfaces or between left and right ends of the wall. The heat transfer has the same direction as the temperature gradient as a result of the temperature change. Both surfaces of the wall are nearly isothermal which means the temperature is the same at almost all points of each surface. The temperature gradient in the direction where the temperature difference occurs, is expected to be large due to the small thickness of the wall. Since there is only heat transfer and no heat generation during this process, the rate of change of the energy of the wall is equal to the rate of heat transfer through the wall which is equal to the difference between the rate of heat transfer into the wall and the rate of heat transfer out of the wall.

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} \quad (1)$$

Heat transfer in this case is steady so $dE_{wall}/dt=0$ because the temperature of the wall remains the same at all points with time. Consequently, the rate of heat transfer through the wall is constant $\left(\dot{Q}_{cond,wall} = constant\right)$ because the rate of heat transfer into the wall becomes equal to rate of heat transfer out of the wall.

Similarly, following the same concept, there is a plane wall whose two surfaces are kept in constant temperatures T_1 and T_2 respectively, with thickness d and average conductivity k. The heat conduction through the wall is steady and one-dimensional. Using Fourier's law for heat conduction we have the following expression:

$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx} \tag{2}$$

The conduction heat transfer rate $\dot{Q}_{cond,wall}$ and the area of the wall A remain constant and so dT=dx remains constant as well. As a result, the temperature changes linearly only in x axis. If we proceed further with the mathematics we get:

$$\int_{x=0}^{d} \dot{Q}_{cond,wall} dx = -\int_{T=T_1}^{T_2} kAdT \rightarrow \dot{Q}_{cond,wall} = -kA\frac{(T_2-T_1)}{d} = kA\frac{T_1-T_2}{d} \tag{3}$$

The final expression shows that the rate of heat conduction through a plane wall is proportional to its thermal conductivity, its area and the temperature difference between its two surfaces but inversely proportional to its thickness. (Young 1994, Çengel 2004, Holman 2008, Sachdeva 2010)

2.2 The concept of thermal resistance including convection and radiation

In heat transfer analysis, steady conditions and surface temperatures are often used when studying heat transfer through a medium for simplicity. These heat transfer problems can be easily solved in proportion to electric circuit problems by introducing thermal resistance concepts instead of using differential equations. In this analysis, thermal resistance, temperature difference, and heat transfer rate correspond to electrical resistance, voltage difference and electric current respectively.

Using Equation (3) that was extracted before for a plane wall, we can make the substitution:

$$R_{cond,wall} = \frac{d}{kA} \tag{4}$$

and obtain a new equation:

$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{cond,wall}} \tag{5}$$

where $R_{cond,wall}$ is the conduction resistance of the wall or in other words the thermal resistance of the wall against heat transfer by conduction. The conduction resistance is proportional to the thickness of the wall and inversely proportional to the area of the surface of the wall and the thermal conductivity of the wall.

The Equation (5) above is similar to the equation below for electric current flow I:

$$I = \frac{V_1 - V_2}{R_e} \tag{6}$$

where, $R_e = {}^{L}/_{\sigma_e A}$ is the electric resistance, σ_e is the electrical conductivity and V_1-V_2 is the voltage difference across the resistance.

If instead of studying only heat transfer though a plane wall, heat transfer from a solid to a fluid is examined, then convection occurs. For a solid surface with an area A_s and temperature T_s and a fluid with temperature T_∞ far from the surface between the two media, Newton's law of cooling is expressed as:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \tag{7}$$

where h is the convection heat transfer coefficient. The electrical analogous would be:

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \tag{8}$$

where,

$$R_{conv} = \frac{1}{hA_s} \tag{9}$$

is the convection resistance of the surface or the thermal resistance of the surface against heat transfer by convection. Under the assumption that h remains constant and uniform, Equation (8) for R_{conv} may be applied for a surface of any geometry. If heat transfer coefficient becomes very large ($h \to \infty$) then $R_{conv} \to 0$, thus there is no resistance to heat transfer by convection. (Young 1994)

When there is a gas on the surrounding space of a wall, radiation effects between the surface of the wall and the surroundings must be taken in account as well. If we consider having a surface with emissivity ε and area A_s at temperature T_s and the average temperature of the surrounding surfaces is T_{surr} the rate of radiation heat transfer rate is written as:

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = \mathrm{h}_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \tag{10}$$

where,

$$R_{rad} = \frac{1}{\mathrm{h}_{rad}\mathrm{A}_s} \tag{11}$$

is the radiation resistance of the surface or the thermal resistance of the surface against heat transfer by radiation. (Young 1994) The radiation heat transfer coefficient h_{rad} is given by the equation:

$$\mathrm{h}_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \tag{12}$$

Both the temperature of the surface T_s and the temperature of the surroundings T_{surr} must be in K for the correct calculation of the radiation heat transfer coefficient h_{rad}.

In the case of a surface exposed to the surrounding air there is heat transfer by both convection and radiation at the same time and both components must be taken in account either by adding them if they are in the same direction or by subtracting them if they are in the opposite direction. The two phenomena may be represented by two resistances, convection resistance and radiation resistance, parallel to each other in a thermal resistance network but the evaluation of mathematical expressions may become complex due to radiation. When the temperature of the surroundings is almost equal to the temperature far enough from the surface, the calculation for the effect of radiation is simplified by introducing a new variable in Equation (9), the combined heat transfer coefficient $h_{combined}$ where:

$$\mathrm{h}_{combined} = \mathrm{h}_{conv} + \mathrm{h}_{rad} \tag{13}$$

(Çengel 2004, Holman 2008, Sachdeva 2010)

2.3 Thermal resistance network

When there is a surface of a wall exposed to ambient air, the general configuration under study is fluid – plane wall – fluid as shown in Figure (1) and there is convection before and after the wall and conduction through the wall. Assuming that heat flow is steady and one-dimensional, the heat transfer rate remains constant through the three media. The plane wall has thickness d, area A and thermal conductivity k. The two surfaces of the wall are isothermal at temperatures T_1 and T_2 with $T_1>T_2$. The fluid before and after the wall are at temperatures $T_{\infty1}$ and $T_{\infty2}$, where $T_{\infty1}>T_{\infty2}$ and have heat transfer coefficients h_1 and h_2 respectively.

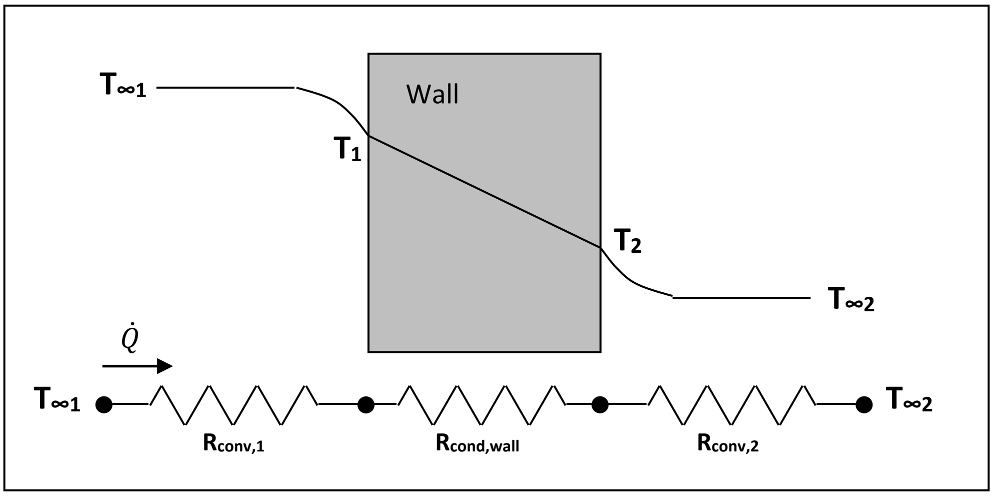

Figure 1: Heat transfer for the configuration fluid – plane wall – fluid and the respective thermal resistance network. (Çengel 2004)

Beginning from $T_{\infty1}$ the temperature drops asymptotically in the fluid while approaching the first surface to T_1, decreases linearly through the wall until the second surface to T_2 and then drops again asymptotically in the fluid while moving away from the second surface to $T_{\infty2}$. The rate of heat transfer balance is written as:

$$\dot{Q} = \mathrm{h}_1 A(T_{\infty1} - T_1) = kA\frac{T_1 - T_2}{d} = \mathrm{h}_2 A(T_2 - T_{\infty2}) \tag{14}$$

Following the mathematical operations we get:

$$\dot{Q} = \frac{T_{\infty1} - T_1}{1/\mathrm{h}_1 A} = \frac{T_1 - T_2}{d/kA} = \frac{T_2 - T_{\infty2}}{1/\mathrm{h}_2 A} = \frac{T_{\infty1} - T_1 + T_1 - T_2 + T_2 - T_{\infty2}}{1/\mathrm{h}_1 A + d/kA + 1/\mathrm{h}_2 A} \rightarrow$$

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} \tag{15}$$

where,

$$R_{total} = R_{conv,1} + R_{cond,wall} + R_{conv,2} = \frac{1}{\mathrm{h}_1 A} + \frac{d}{kA} + \frac{1}{\mathrm{h}_2 A} \tag{16}$$

is the total thermal resistance and is the equivalent resistance found by the addition of the three individual resistances which are in series as in an electric circuit. The final expression (Equation (15)) clearly shows that the rate of heat transfer for this configuration under steady conditions is proportional to the temperature difference and inversely proportional to the total thermal resistance. Another observation that can be made is that the temperatures on each surface of the wall are not needed for the evaluation of the rate of heat transfer, only the temperatures of the fluids before and after the wall are used in the calculation. It can also be noted that, since the rate of heat transfer remains constant at all segments of the configuration above, the temperature difference at any segment is proportional to the respective thermal resistance. In other words, temperature drop at any segment is equal to the thermal resistance multiplied by the constant rate of heat transfer,

$$\Delta T = \dot{Q} R \qquad (17)$$

Using Newton's law of cooling, heat transfer could be expressed as:

$$\dot{Q} = UA\Delta T \qquad (18)$$

where U is the overall heat transfer coefficient. If we take Equation (15) and Equation (18) we obtain the following equation:

$$UA = \frac{1}{R_{total}} \qquad (19)$$

which shows that if we consider having a unit area (A=1 m^2) the overall heat transfer coefficient is equal and inversely proportional to the total thermal resistance.
(Çengel 2004)

2.4 Multilayer plane walls

At most of the times, each wall of a building is not made of only one homogeneous material but instead it is more composite consisting of various layers of different materials and each one has different thickness d and thermal conductivity k while the surface area A remains the same for all the materials in the case of a plane wall. Before and after the wall there is a fluid with temperatures $T_{\infty 1}$ and $T_{\infty 2}$ and usually it is air. The aim of this master thesis is the calculation of the overall heat transfer coefficient for such complex building elements.

Again, the thermal resistance concept can be used for the calculation of steady one-dimensional heat transfer. The electrical analogy in this case using the thermal resistance network is to consider the total thermal resistance of the wall as the sum of the individual thermal resistances of all the materials connected in series in the direction where the temperature drop occurs. Each material has conductive resistance equal to d/kA.

The total thermal resistance R_{total} for a multilayer plane wall consisting of n layers is expressed as:

$$R_{total} = R_{conv,1} + R_{cond,wall\,1} + \cdots + R_{cond,wall\,n} + R_{conv,2} \rightarrow$$

$$R_{total} = \frac{1}{h_1 A} + \frac{1}{A}\sum_i^n \frac{d_i}{k_i} + \frac{1}{h_2 A} \tag{20}$$

The rate of heat transfer $\dot{Q}$ could be calculated for a composite wall as well, using Equation (15). If the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ and the rate of heat transfer $\dot{Q}$ are known, the temperature on the interface between two media (fluid-wall or wall-fluid) or between two layers of the wall can be determined. (Çengel 2004)

2.5 Calculation of thermal values according to ISO 6946 and ISO 10456

As mentioned in standard ISO 6946:2007 a building element is a "major part of a building such as a wall, floor or roof" while a building component is described as a "building element or part of it". For the purpose of this master thesis, emphasis will be given only on the calculation of thermal resistance and transmittance of the roof, the wall and the floor, which are building configurations (components and/or elements) that do not exchange heat with the ground and are not designed to be permeated with air such as doors, windows and curtain walling. Every thermally homogeneous part of a component has its own unique thermal resistance. Using all the thermal resistances of the materials that form the studied component and by taking in account the effect of the internal surface resistance and the external surface resistance, the total thermal resistance of the component can be calculated. The internal and the external thermal resistance occur due to the internal and the external environment of the component/element respectively. (EN ISO 6946:2007)

If the design thermal conductivity value, k, and the respective thickness, d, of a material in the component are known, the design thermal resistance value R for a unit area A can be calculated using the equation:

$$R = \frac{d}{k} \quad (21)$$

When there aren't any available data to describe the boundary conditions, for plane surfaces the external surface resistance R_{se} is always 0.04 (m^{2}*^{o}C)/W and is independent of the direction of heat flow. The internal surface resistance value though, varies according to the direction of heat flow. For upward heat flow, R_{si} equals to 0.10 (m^{2}*^{o}C)/W, for horizontal heat flow R_{si} is 0.13 (m^{2}*^{o}C)/W and for downward heat flow R_{si} is 0.17 (m^{2}*^{o}C)/W. As a result, the internal surface resistance changes for every building element involved and takes values between 0.10 for the roof, 0.13 for a wall and 0.17 for the floor. The concept of using surface resistances is valid only for surfaces in contact with air and no other material. The values are summarised in Table (1). (EN ISO 6946:2007)

When a planar element consists of n thermally homogeneous materials perpendicular to the heat flow, each layer has its own unique design thermal resistance. The total thermal resistance R_T is thus given by the expression:

$$R_T = R_{si} + R_1 + R_2 + \cdots R_n + R_{se} \quad (22)$$

The thermal transmittance, or overall heat transfer coefficient or U-value is therefore given by:

$$U = \frac{1}{R_T} \quad (23)$$

If Equations (21),(22),(23) are combined, the following equation is extracted, which correlates the thermal transmittance U of a building element, with the internal surface resistance R_{si}, the external surface resistance R_{se} and the thickness d and thermal conductivity k of every material included in the building element. Every material is a thermally homogeneous layer perpendicular to the heat flow.

$$U = \frac{1}{R_{si} + \sum_i^n \frac{d_i}{k_i} + R_{se}} \quad (24)$$

Building Element	Internal Thermal Resistance R_{si} [(m^2*°C)/W]	External Thermal Resistance R_{se} [(m^2*°C)/W]
Roof	0.10	0.04
Wall	0.13	
Floor	0.17	

Table 1: Internal and external thermal resistance of building elements (EN ISO 6946:2007)

Material	Thermal Conductivity k [W/(m*°C)}
Brick	0.4
Perforated thermal brick	0.2
Plaster	1
Insulating material (polystyrene)	0.03
Mineral wool	0.041
Screed	1.35
Reinforced Concrete	2.4
Plasterboard	0.25
Air	R = 0.18 [(m^2*°C)/W]
Wood board	0.15
Trowel	0.8
Waterproofing layer	0.23

Table 2: Thermal Conductivity k of various materials (EN ISO 10456:2007, Fokaides)

According to standard ISO 10456:2007 a material with certain dimensions and shape, covered with facings or coatings is in its final form ready to be used and forms a product. Reciprocally, a product without a specific shape, dimensions and delivery form, not covered with any facings or coatings and irrespectively of its final form is defined as a material. In order to describe the typical performance of a product or a material when it is incorporated in a building component, the value of its design thermal resistance or conductivity is used. The design thermal resistance or conductivity value is obtained under specific internal and external conditions. Design values can be found using a particular set of conditions such as a reference temperature, the moisture content and the ageing of a material. Also they can be measured with methods explained in detail in the standard ISO 10456:2007 or they can be calculated indirectly using values of related properties from tables. It is advisable though to use the manufacturer's values for a material when available, instead of calculated or measured data. (EN ISO 10456:2007)

The materials that will be used at a later stage in this master thesis are shown in Table (2) with their thermal conductivity value k.

2.6 Directive 2010/31/EU on the energy performance of buildings

Article 5 of the European Directive 2010/31 focuses on the "calculation of cost-optimal levels of minimum energy performance requirements". It suggested the establishment of a methodology framework by 30 June 2011 as a basis for comparisons about buildings and building elements so as to calculate the cost-optimal levels of their minimum energy performance requirements. The cost-optimal level is defined as "the energy performance level which leads to the lowest cost during the estimated economic lifecycle". The aforementioned framework should comply with Annex III –that will be analyzed later on- of the same Directive and should include variations on the suggested methodology for different categories of buildings and especially between new and existing buildings.

Member States should make the necessary calculations of the cost-optimal levels of the minimum energy performance requirements based on the methodology framework already set, by taking in account weather conditions and the convenience to access infrastructure relevant to energy. Then, the calculated results should be compared to the currently effective minimum energy performance requirements. Beginning from 30 June 2012, Member States should prepare a report and submit it to the European Commission regularly, at intervals not larger than five years that would include all the input data and assumptions used to perform the calculations and the results extracted by them.

If the comparison done before had showed that the minimum energy performance requirements in effect are notably less energy efficient than the cost-optimal levels of minimum energy performance requirements, the affected Member State should give explanations to the report submitted to the Commission and also present a plan to decrease the difference. The Commission on its turn should publish a report showing the progress of the Member States in complying with the targets set in Article 5 of the Directive 2010/31.

Annex III explains how the comparative methodology framework should be used by the Member States effectively in order to be able to define the energy performance of buildings and building elements, identify the economic aspects occurring from energy performance measures and correlate them so as to find the cost-optimal level. It is also stated that detailed guidelines, given by the Commission, should accompany the framework to assist in its application to calculate the cost-optimal performance levels. The framework should be based on European standards relating to Directive 2010/31 and should be able to account climatic conditions, costs of investments, category of a building, mode of usage of a building, earnings from production of energy if there are any etc. The Commission should also provide information regarding estimated developments on the energy price in the long-term.

Member States in accordance with the comparative methodology framework should firstly identify buildings that are representative of their functionality, geographic location and climatic conditions indoors and outdoors as reference buildings. Then, they should decide measures for energy efficiency that would be evaluated on the reference buildings. Afterwards, the final and primary energy need of the reference buildings as well as the reference buildings after the application of the energy efficiency measures should be assessed. Next, the costs of the energy efficiency measures that were applied on the reference buildings should be calculated for their expected economic lifecycle following the framework principles. The Member States should then assess how cost-effective the different levels of minimum energy performance requirements are and determine the cost-optimal levels of energy performance requirements. (DIRECTIVE 2010/31/EU)

Chapter 3
Methodology, Results and Discussion

In this chapter, firstly the methodology followed to make all the necessary calculations for the six building elements is explained. Then, using two Matlab codes the overall heat transfer coefficient is calculated and its dependence on the thickness of insulation is presented graphically for every building element under study. Finally, comparisons between the results of the building elements are done and also there is an effort to estimate the optimum insulation thickness for each one.

3.1 Methodology

The purpose of this master thesis was to develop a numerical application for the calculation of thermal transmittance of building elements. For this reason a code was written in Matlab programming package. The code **projectheattransfer.m** initially uses data from two text files as input. The first file **rescoef.dat** (data shown on Table (1)) provides the internal thermal resistance according to the choice of the building element by the user between roof, wall and floor.

The second file **thermcond.dat** (data shown on Table (2)) stores the thermal conductivities of some materials used in the construction of the different building elements. Those materials are brick, perforated, thermal brick, plaster, polystyrene (insulating material), mineral wool (also insulating material), screed, reinforced concrete, plasterboard, air, wood board, trowel and waterproofing layer. The user chooses the number of materials which the building element consists of, the type of the materials and their thickness. The code then calculates the total thermal resistance of the building element as a sum of the thermal resistance of every material. The thermal resistance of every individual material is found by dividing its thickness with the thermal conductivity. Exemption was made for the air whose thermal resistance value was directly used.

Afterwards, the code uses Equation (24) to calculate the thermal transmittance of the building element by taking in account the internal surface resistance, the external surface resistance and the total thermal resistance of the building element. The numerical values of the calculated thermal transmittance in relation to the thickness of insulation are then saved in a separate text file for every building element under study.

Six different building elements were chosen. Three external walls, two roofs and one column which is accounted again as external wall. The materials are inserted in the code starting from the internal and moving outwards. All the materials are homogeneous plane surfaces. The details of every building element are shown in Tables (3, 5, 7, 9, 11, 13).

In general, one insulating material was used in every building element, either polystyrene or mineral wool. Firstly, the thermal transmittance of every building element was calculated without any insulation and then the thickness of insulation was being increased up to 14 cm with a step of 2 cm and the corresponding thermal transmittance was calculated each time. The calculated values are summarised in Tables (4, 6, 8, 10, 12, 14).

For the graphical representation of the results, another code was implemented in Matlab, saved as **plot_code.m**. In this code, the six text files (**results1…6.txt**) with the calculations that were previously created are used as input and for every building element a graph is created to show the dependence of thermal transmittance on the thickness of the insulation. The results are presented graphically in Figures (2-7). Next, two more graphs were created to compare the drop of thermal transmittance with an increasing insulation thickness between the different configurations for the same building elements, one for the external walls, Figure (8) and one for the roofs, Figure (9). A summary table was also created to show the consolidated data of the thickness of insulation with the respective calculated thermal transmittance and the percentage difference between consecutive U-values for all the building elements.

3.2 Results

<u>Building Element Nº 1</u>:

External Wall consisting of 4 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Plaster	0.025
Brick	0.2
Insulating Material (polystyrene)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Plaster	0.025

Table 3: The materials which building element Nº 1 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	1.388889
0.02	0.721154
0.04	0.487013
0.06	0.367647
0.08	0.295276
0.10	0.246711
0.12	0.211864
0.14	0.185644

Table 4: Thickness of insulation used in building element Nº 1 and the respective calculated thermal transmittance.

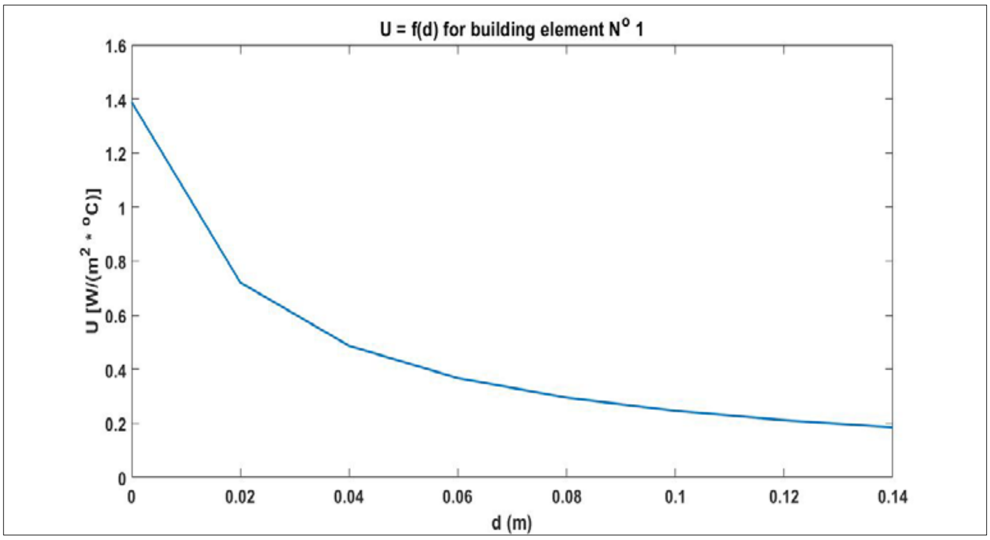

Figure 2: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 1.

Building Element Nº 2:

External Wall consisting of 4 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Plaster	0.025
Perforated Thermal Brick	0.3
Insulating Material (polystyrene)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Plaster	0.025

Table 5: The materials which building element Nº 2 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	0.581395
0.02	0.418994
0.04	0.327511
0.06	0.268817
0.08	0.227964
0.10	0.197889
0.12	0.174825
0.14	0.156576

Table 6: Thickness of insulation used in building element Nº 2 and the respective calculated thermal transmittance.

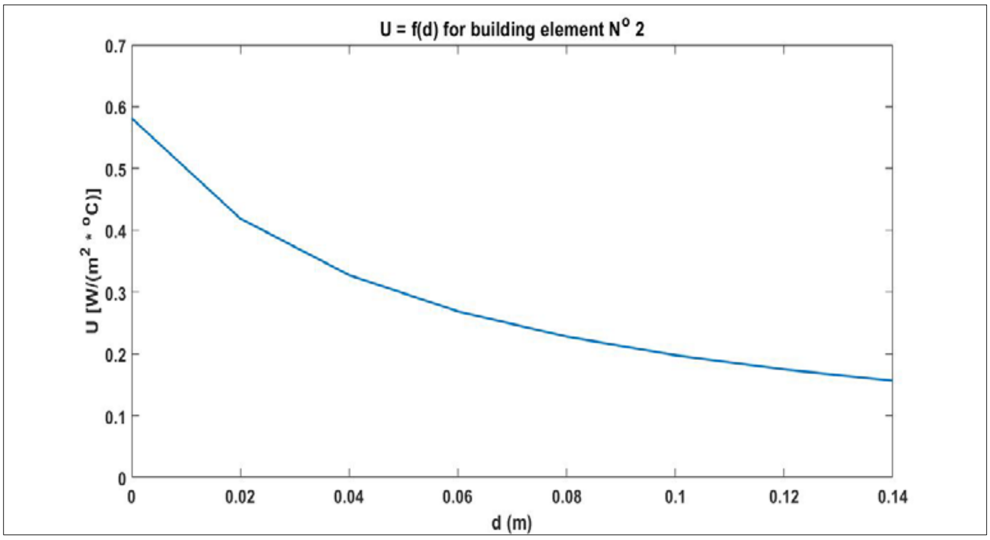

Figure 3: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 2.

<u>Building Element Nº 3</u>:

Roof consisting of 5 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Trowel	0.01
Reinforced Concrete	0.15
Mineral Wool (Insulating Material)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Screed	0.1
Waterproofing layer	0.005

Table 7: The materials which building element Nº 3 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	3.217367
0.02	1.023039
0.04	0.608218
0.06	0.432748
0.08	0.335854
0.10	0.274413
0.12	0.231975
0.14	0.200905

Table 8: Thickness of insulation used in building element Nº 3 and the respective calculated thermal transmittance.

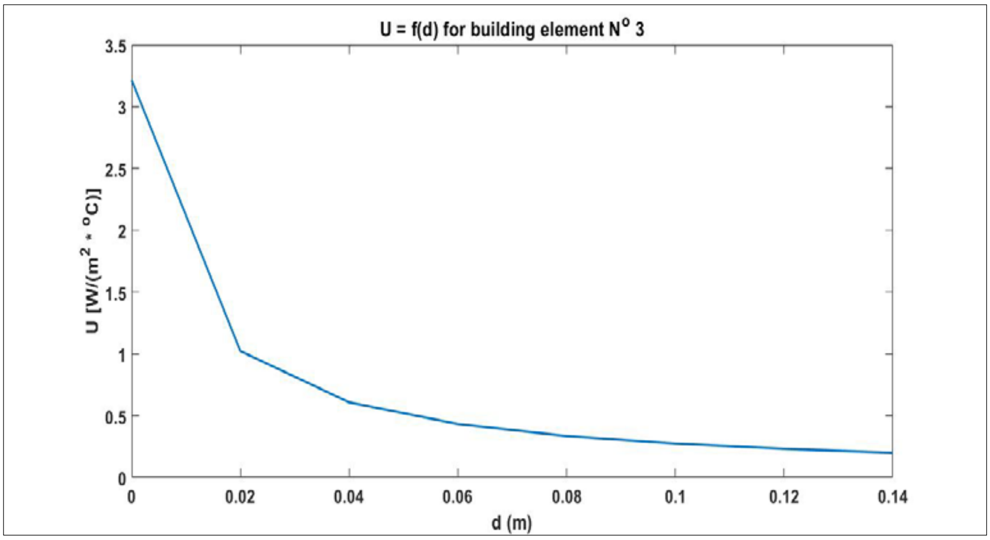

Figure 4: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 3.

Building Element Nº 4:

Roof consisting of 5 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Plasterboard	0.012
Air	
Mineral Wool (Insulating Material)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Reinforced Concrete	0.2
Screed	0.05

Table 9: The materials which building element Nº 4 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	2.047626
0.02	1.024406
0.04	0.683069
0.06	0.512352
0.08	0.409905
0.10	0.341601
0.12	0.292809
0.14	0.256213

Table 10: Thickness of insulation used in building element Nº 4 and the respective calculated thermal transmittance.

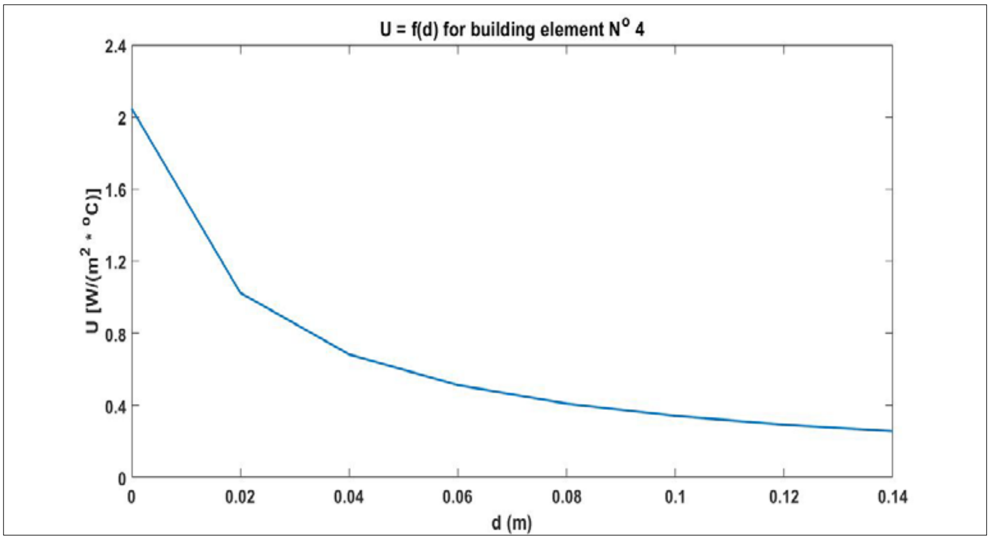

Figure 5: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 4.

Building Element Nº 5:

External Wall consisting of 6 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Plasterboard	0.012
Mineral Wool (Insulating Material)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Air	
Brick	0.2
Plaster	0.025
Wood board HTP	0.05

Table 11: The materials which building element Nº 5 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	0.795967
0.02	0.573349
0.04	0.448040
0.06	0.367681
0.08	0.311764
0.10	0.270610
0.12	0.239054
0.14	0.214088

Table 12: Thickness of insulation used in building element Nº 5 and the respective calculated thermal transmittance.

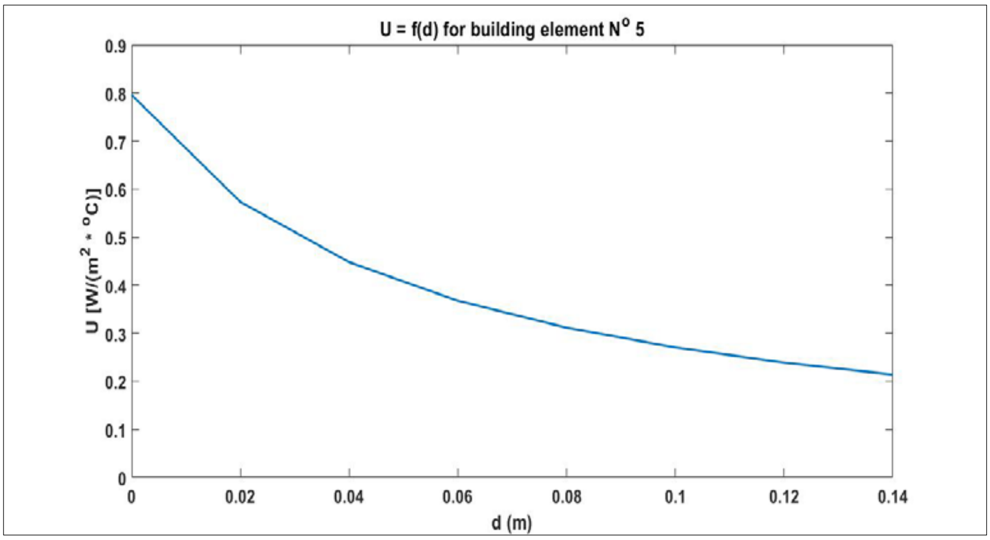

Figure 6: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 5.

<u>Building Element Nº 6</u>:

Column (External Wall) consisting of 5 materials (Fokaides, Fokaides 2014)

Material	**Thickness d (m)**
Plasterboard	0.012
Mineral Wool (Insulating Material)	0 0.02 0.04 0.06 0.08 0.10 0.12 0.14
Air	
Reinforced Concrete	0.25
Plaster	0.025

Table 13: The materials which building element Nº 6 consists (starting from the internal) of and their respective thickness.

Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**
0.00	1.896933
0.02	0.985249
0.04	0.665435
0.06	0.502366
0.08	0.403488
0.10	0.337133
0.12	0.289520
0.14	0.253691

Table 14: Thickness of insulation used in building element Nº 6 and the respective calculated thermal transmittance.

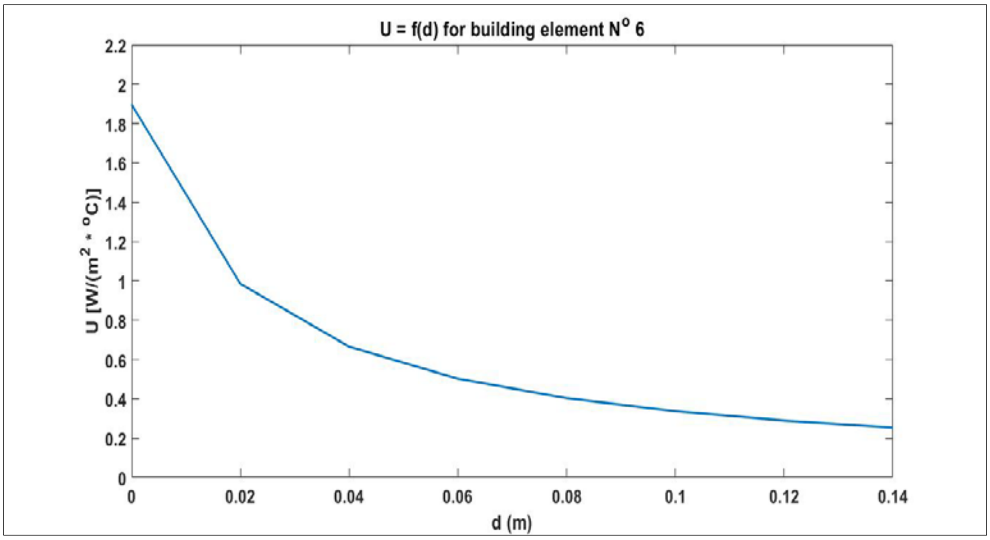

Figure 7: Dependence of thermal transmittance on the thickness of the insulation for the building element Nº 6.

Building Element Nº 1			Building Element Nº 2		
Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**	**Thickness of Insulation d (m)**	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**
0.00	1.388889		0.00	0.581395	
0.02	0.721154	-48.08%	0.02	0.418994	-27.93%
0.04	0.487013	-32.47%	0.04	0.327511	-21.83%
0.06	0.367647	-24.51%	**0.06**	**0.268817**	**-17.92%**
0.08	**0.295276**	**-19.68%**	0.08	0.227964	-15.20%
0.10	0.246711	-16.45%	0.10	0.197889	-13.19%
0.12	0.211864	-14.12%	0.12	0.174825	-11.66%
0.14	0.185644	-12.38%	0.14	0.156576	-10.44%

Building Element Nº 3			Building Element Nº 4		
Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**	**Thickness of Insulation d (m)**	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**
0.00	3.217367		0.00	2.047626	
0.02	1.023039	-68.20%	0.02	1.024406	-49.97%
0.04	0.608218	-40.55%	0.04	0.683069	-33.32%
0.06	0.432748	-28.85%	0.06	0.512352	-24.99%
0.08	0.335854	-22.39%	0.08	0.409905	-20.00%
0.10	**0.274413**	**-18.29%**	**0.10**	**0.341601**	**-16.66%**
0.12	0.231975	-15.47%	0.12	0.292809	-14.28%
0.14	0.200905	-13.39%	0.14	0.256213	-12.50%

Building Element Nº 5			Building Element Nº 6		
Thickness of Insulation d (m)	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**	**Thickness of Insulation d (m)**	**Thermal Transmittance U (W/(m²*ºC))**	**Percentage Difference between consecutive U-values**
0.00	0.795967		0.00	1.896933	
0.02	0.573349	-27.97%	0.02	0.985249	-48.06%
0.04	0.448040	-21.86%	0.04	0.665435	-32.46%
0.06	**0.367681**	**-17.94%**	0.06	0.502366	-24.51%
0.08	0.311764	-15.21%	**0.08**	**0.403488**	**-19.68%**
0.10	0.270610	-13.20%	0.10	0.337133	-16.45%
0.12	0.239054	-11.66%	0.12	0.289520	-14.12%
0.14	0.214088	-10.44%	0.14	0.253691	-12.38%

Table 15: Summary of thickness of insulation with the respective calculated thermal transmittance and the percentage difference between consecutive U-values for all the building elements.

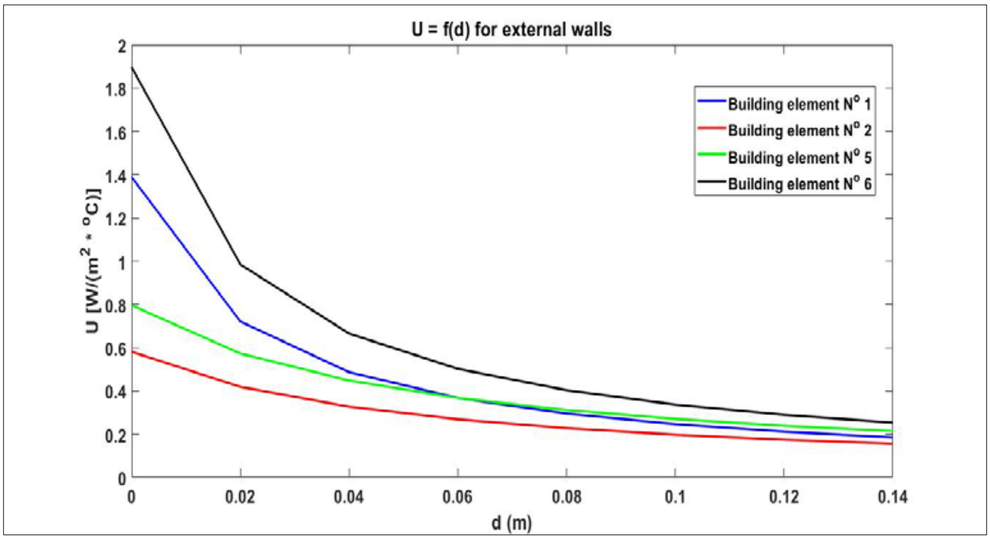

Figure 8: Dependence of thermal transmittance on the thickness of the insulation for the building elements Nο 1, Nο 2, Nο 5 and Nο 6.

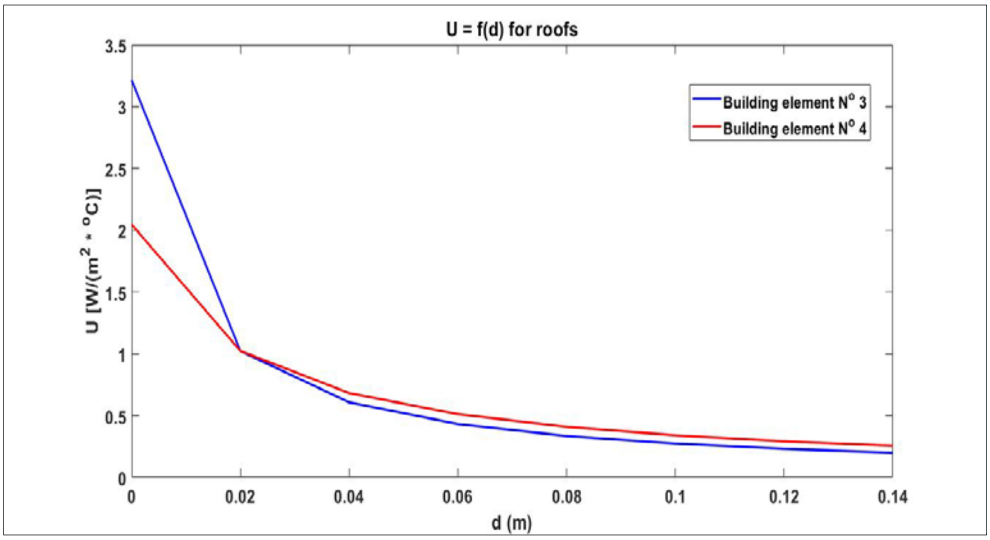

Figure 9: Dependence of thermal transmittance on the thickness of the insulation for the building elements Nο 3 and Nο 4.

3.3 Discussion

Six building elements were studied in this master thesis, three external walls, two roofs and one column which may be classified in the former category. Building elements Nο 1, Nο 2, Nο 5 & Nο 6 were external walls and building elements Nο 3 & Nο 4 consisted of roofs. As a general result from all the building elements, thermal transmittance initially decreased with the inclusion of thermal insulation and continued to diminish asymptotically with the increase of the insulation thickness as shown in Figures (2-7). Next, one observation that can be made from Figures (8 & 9) is that the higher the initial value of thermal transmittance of a building element without insulation was, the bigger was the drop once insulation was added in the configuration. As insulation was increased, the rate of decrease of thermal transmittance was higher for the building elements which initially had a higher value of thermal transmittance.

Also, it can be clearly seen from all the Figures that for very large values of insulation thickness, thermal transmittance can theoretically approach zero but this is not practical or feasible in real life applications because it would cost a lot of money during the construction or renovation of a building. The range of lowest values of thermal transmittance calculated for all the building elements was approximately U=0.16-0.26 $W/(m^2*^oC)$ for insulation thickness=0.14 m. From a point and further though, it is noticed that even though insulation increases, thermal transmittance decreases insignificantly. For this reason, an effort was done to estimate the optimum insulation thickness qualitatively. Firstly, the percentage difference between two consecutive values of U was calculated using Equation (25) and all the values were recorded in Table (15).

$$\text{Percentage difference} = \frac{U_{i+1} - U_i}{U_i} * 100\% \quad (25)$$

Then, when this difference dropped lower than 20% the reciprocal thickness of insulation was identified as the optimum value. For the studied building elements N° 1, N° 2, N° 3, N° 4, N° 5 and N° 6, the optimum insulation thickness was found to be 0.08 m, 0.06 m, 0.10 m, 0.10 m, 0.06 m and 0.08 m respectively (N° 1, N° 2, N° 5 & N° 6 correspond to external walls and N° 3 & N° 4 correspond to roofs).

If we compare the configurations between the first and the second building elements, their only difference is that in the former brick was used while in the latter perforated thermal brick was selected with bigger thickness but with substantially lower thermal conductivity value. As a result, the relevant thermal transmittance values of the second building element were substantially lower than the first building element for insulation thickness up to 0.06 m as shown in Figure (8). The perforated thermal brick is therefore a much better choice when building a wall because the aim is to save energy. At both building elements polystyrene was used as the insulating material.

The fifth building element was a more complex structure with mineral wool for insulation and space of air inside the wall. It also contained wood board, a material with low thermal conductivity. It performed better than the first building element for insulation thickness up to 0.06 m and for greater thickness of insulation both building elements gave almost the same results as shown in Figure (8). Its performance though was slightly worse than the second building element. The sixth building element consisted of plasterboard, mineral wool and space of air internally as the fifth building element but instead of brick, plaster and wood board, it contained reinforced concrete and plaster for the columns of a building and gave the highest set of values for the thermal transmittance.

The third and fourth building elements were roofs with different composition. Both of them included mineral wool for insulation and the fourth element also contained space of air in the construction. The fourth building element gave much lower values for the thermal transmittance than the third building element for insulation thickness up to 0.02 m while for greater insulation thickness both elements had almost identical values for the overall heat transfer coefficient as shown in Figure (9).

In conclusion, the second building element and the fourth building element had the best performance among the walls and roofs respectively.

Chapter 4
Conclusions

In this master thesis, a programming code was implemented in Matlab programming package so as to evaluate the effect of thermal insulation on the thermal transmittance of six building elements. The building elements studied included roofs and external walls and all of them consisted of multiple layers of different materials with different thermal characteristics and thickness. All the building elements were treated as plane surfaces and all the calculations were performed on the basis of a unit area. The outer surface of all the building elements was exposed to the open air externally and their inner surface was in direct contact with air internally. No specific boundary information was known, e.g. indoor and outdoor air temperatures or temperatures on the inner and outer surface of each building element. For this reason, the values of internal surface resistance and external surface resistance were used.

The composition of each element was alternating from one another and so the effect on the total thermal resistance (R-value) and subsequently on the thermal transmittance (U-value) was different for each case and was initially studied without the existence of any insulation. The calculation of the total thermal resistance of each building element was calculated by adding the individual thermal resistance of each material based on the thermal resistance network concept. Next, insulation was included and it was clearly shown for all the configurations that for increasing insulation thickness, heat transfer rate through the building element, roof or external wall, decreases. Insulation is very important in the building sector because it is used to make buildings more energy saving and thus less energy is needed to cover the needs of a building for heating and cooling. Also less energy is lost in the environment during winter or is absorbed from it in summer. Moreover, fewer natural resources are needed for the production of this energy and less money is spent for the consumption of this energy.

The numerical application that was implemented in this master thesis is a good basis, in general, for the study of heat transfer in building elements but it works adequately well for the evaluation of steady one-dimensional heat transfer only in plane building elements consisting of a series of materials that are perpendicular to the heat flow. Further improvements could be done in order to be able to use the code for more complex configurations which may consist of a combination of materials perpendicular and/or parallel to the heat flow. The electrical analogy using the thermal resistance network in this case would be to represent the materials parallel to heat flow with resistances in parallel arrangement and the materials perpendicular to heat flow as resistances in series and perform the necessary calculations to find the equivalent total thermal resistance of the composite building elements. Also, the code could be enhanced so as to apply for other geometries except of large plane walls such as long cylinders and spheres that may be approached again as one-dimensional and it is quite simple to obtain an analytical solution. The code could not be used though for more complex geometries in real life problems, either two-dimensional or three-dimensional where there is no analytical solution available.

Appendix A
First Matlab Code

```
Re=0.04;
m=1;
U=0;
DK=0;
x=0;

file1 = fopen('rescoef.dat');
A = textscan(file1,'%s %f');
fclose(file1);

element=A{1};
Ri=A{2};

prompt1='Choose the building element. For roof press 1, for wall press 2 or
for floor press 3:\n';
optiona=input(prompt1);
fprintf('\n');
Rii=Ri(optiona);
fprintf('The chosen building element is: ');
disp(element(optiona));
fprintf('Its internal surface resistance Ri in (m^2*oC)/W is: %f\n',Rii);
fprintf('\n');
fprintf('The external surface resistance Re in (m^2*oC)/W is: %f\n',Re);
fprintf('\n');

file2 = fopen('thermcond.dat');
B = textscan(file2,'%s %f');
fclose(file2);

mat=B{1};
k=B{2};

prompt2='Choose the number of materials of the building element:\n';
optionb=input(prompt2);
fprintf('\n');
fprintf('Give the materials of the building element in the order which they
appear.\n');
fprintf('For every material press the corresponding number.\n');
fprintf('\n');
fprintf('----------------------------------------------\n');
fprintf('I   Brick -----------------------------  1   I\n');
fprintf('I   Perforated thermal brick ----------  2   I\n');
fprintf('I   Plaster ---------------------------  3   I\n');
fprintf('I   Insulating material (polystyrene) -  4   I\n');
fprintf('I   Mineral wool ----------------------  5   I\n');
fprintf('I   Screed ----------------------------  6   I\n');
fprintf('I   Reinforced concrete ---------------  7   I\n');
fprintf('I   Plasterboard ----------------------  8   I\n');
fprintf('I   Air -------------------------------  9   I\n');
fprintf('I   Wood board ------------------------ 10   I\n');
fprintf('I   Trowel ---------------------------- 11   I\n');
fprintf('I   Waterproofing layer --------------- 12   I\n');
fprintf('----------------------------------------------\n');
fprintf('\n');
```

```
fprintf('For every material give also its thickness.\n');
fprintf('\n');

while m<=optionb
    fprintf('Material No: %d \n',m);
    fprintf('--------------- \n');
    prompt3='Choose the material:\n';
    optionc=input(prompt3);
    kk=k(optionc);
    material=mat(optionc);
    fprintf('\n');
    fprintf('The material is: ');
    disp(material);
    if optionc==9
        fprintf('Its thermal resistance R in (m^2*oC)/W is: %f\n',kk);
        fprintf('\n');
        x=kk;
    else
    fprintf('Its thermal conductivity k in W/(m*oC) is: %f\n',kk);
    fprintf('\n');
    prompt4='Choose the thickness d in m:\n';
    d=input(prompt4);
    fprintf('\n');
    x=d/kk;
    end

    DK=DK+x;
    m=m+1;
end

U=1/(Rii+DK+Re);
fprintf('The thermal transmittance U-value of the building element in
W/(m^2*oC)is: %f\n',U);
```

Appendix B
Second Matlab Code

```
file1 = fopen('results1.txt');
A = textscan(file1,'%f %f');
fclose(file1);

d1=A{1};
U1=A{2};

figure(1);
plot(d1,U1,'LineWidth',2);
title('U = f(d) for building element N^o 1');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

file2 = fopen('results2.txt');
B = textscan(file2,'%f %f');
fclose(file2);

d2=B{1};
U2=B{2};

figure(2);
plot(d2,U2,'LineWidth',2);
title('U = f(d) for building element N^o 2');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

file3 = fopen('results3.txt');
C = textscan(file3,'%f %f');
fclose(file3);

d3=C{1};
U3=C{2};

figure(3);
plot(d3,U3,'LineWidth',2);
title('U = f(d) for building element N^o 3');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

file4 = fopen('results4.txt');
D = textscan(file4,'%f %f');
fclose(file4);

d4=D{1};
U4=D{2};

figure(4);
plot(d4,U4,'LineWidth',2);
title('U = f(d) for building element N^o 4');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');
```

```
file5 = fopen('results5.txt');
E = textscan(file5,'%f %f');
fclose(file5);

d5=E{1};
U5=E{2};

figure(5);
plot(d5,U5,'LineWidth',2);
title('U = f(d) for building element N^o 5');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

file6 = fopen('results6.txt');
G = textscan(file6,'%f %f');
fclose(file6);

d6=G{1};
U6=G{2};

figure(6);
plot(d6,U6,'LineWidth',2);
title('U = f(d) for building element N^o 6');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

figure(7);
plot(d1,U1,'b',d2,U2,'r',d5,U5,'g',d6,U6,'k','LineWidth',2);
title('U = f(d) for external walls');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');

figure(8);
plot(d3,U3,'b',d4,U4,'r','LineWidth',2);
title('U = f(d) for roofs');
xlabel('d (m)');
ylabel('U (W/(m^2 * ^oC))');
```

Bibliography

Çengel, Yunus A. (2004). *Heat transfer: a practical approach*, 2nd edition, New York: McGraw-Hill, Chapter 3: Steady Heat Conduction, pp. 127-207.

DIRECTIVE 2010/31/EU OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 19 May 2010 on the energy performance of buildings (recast), 18.6.2010, *Official Journal of the European Union*, pp. 13-35.

EN ISO 6946:2007 Building components and building elements – Thermal resistance and thermal transmittance – Calculation method.

EN ISO 10456:2007 Building materials and products – Hygrothermal properties – Tabulated design values and procedures for determining declared and design thermal values.

Fokaides, P. A., for Nicolaides&Kountouris Metal Company Ltd, Powerpoint presentation, "Ενεργειακή Απόδοση Βιομηχανικών Κτηρίων", Υπολογισμός U Value δομικών στοιχείων (βάσει ISO 6946:2017), Sustainable Energy Research Group.

Fokaides, P. A., Christoforou, E. A., & Kalogirou, S. A. (2014). Legislation driven scenarios based on recent construction advancements towards the achievement of nearly zero energy dwellings in the southern European country of Cyprus. *Energy*, 66, pp. 588-597.

Holman, J. P. (2008). *Heat Transfer*, 9th edition, New Delhi: Tata McGraw Hill Book Co.

Sachdeva, R. C. (2010). *Fundamentals of Engineering Heat and Mass Transfer*, 4th edition, New Age International Publishers.

Tipperary Energy Agency, Module 3.2 Resistance of air layers and surface layers, http://tea.ie/wp-content/uploads/2011/09/Module-3.2-Resistance-of-air-layers-and-surface-layers.pdf. [Retrieved on 30/11/2019].

Young, Hugh D. (1994). *University Physics, Volume A: Mechanics and Thermodynamics*, 8th edition, Addison-Wesley, Chapter 15: Temperature and Heat, pp. 414-449.